

# Reliability of double-wall containment against the impact of hard projectiles



Nadeem A. Siddiqui\*, Baha M.A. Khateeb, Tarek H. Almusallam, Husain Abbas

Department of Civil Engineering, King Saud University, Riyadh 11421, Saudi Arabia

## HIGHLIGHTS

- The reliability of double-wall containment against impact of projectiles was studied.
- Probabilistic procedure based on Monte Carlo simulation technique was used.
- Sensitivity studies were carried out to obtain the results of practical interest.
- Reliability is correlated with the ballistic limit of the outer RC wall.

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## ABSTRACT

Effectiveness of single or double-wall containment structures against a possible strike of projectiles, missiles or airplanes is well researched. However, how the uncertainties involved in the various design parameters influence the reliability of the containment is not very well known. In a double-wall containment structure, as name implies, there are two walls – an outer thick reinforced concrete (RC) wall and an inner thin steel shell/wall. In the present study, a simple probabilistic procedure based on Monte Carlo simulation technique is presented to study the reliability of double-wall containment structures against the impact of external hard projectiles on outer RC wall for varying impact velocities. In order to illustrate the proposed methodology, an idealized double-wall containment structure and a hard projectile were chosen. The probability of failure and the reliability indices of the selected double-wall containment structure were obtained for different striking velocities of the projectile and safety of the containment was correlated with the ballistic limit of the outer RC wall. The results of the study show that the double-wall containment is “safe enough” against the impact of the selected projectile if the projectile nominal velocity is less than 65% of the containment outer wall's nominal ballistic limit ( $V_{BL}$ ). Results also show that under the given uncertainties, if the nominal impact velocity is less than 65% of the nominal ballistic limit of the outer RC wall (i.e.  $0.65V_{BL}$ ), failure probability of the containment is almost zero. However, when impact velocity is more than  $0.90V_{BL}$ , failure probability of the double-wall containment is quite high. It was also observed that a little change in the impact velocity over  $0.90V_{BL}$  may cause a phenomenal change in the containment reliability due to substantial change in the residual kinetic energy of the projectile. A number of sensitivity studies have also been carried out to obtain the results of practical interest.

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## 1. Introduction

A number of single and double-wall containment structures are in existence all over the world. In single-wall containment (Abbas et al., 1996; Siddiqui et al., 2003), a single reinforced concrete (RC) or prestressed wall surrounds the process equipment and machines and provides protection to them against the effects of the

outer/external environmental forces or the release of radiations to the environment. In a double-wall containment structure, as name implies, there are two walls – an outer relatively thick RC wall and an inner steel wall. The outer RC wall of the containment (Fig. 1) gives the external protection to the inner steel wall (which may house the reactor, steam generators and other vital plant equipments) against the effects of the outer environment and impacts of external missiles, projectiles and even aircrafts. The purpose of the inner steel wall/shell is to prevent the emissions in the event of a process failure.

Despite the existence of a large number of containment structures, there are only a few containment structures whose detailed

\* Corresponding author. Tel.: +966 14676962; fax: +966 14677008.

E-mail addresses: [nadeem@ksu.edu.sa](mailto:nadeem@ksu.edu.sa), [nadeemibnehashim@gmail.com](mailto:nadeemibnehashim@gmail.com) (N.A. Siddiqui).

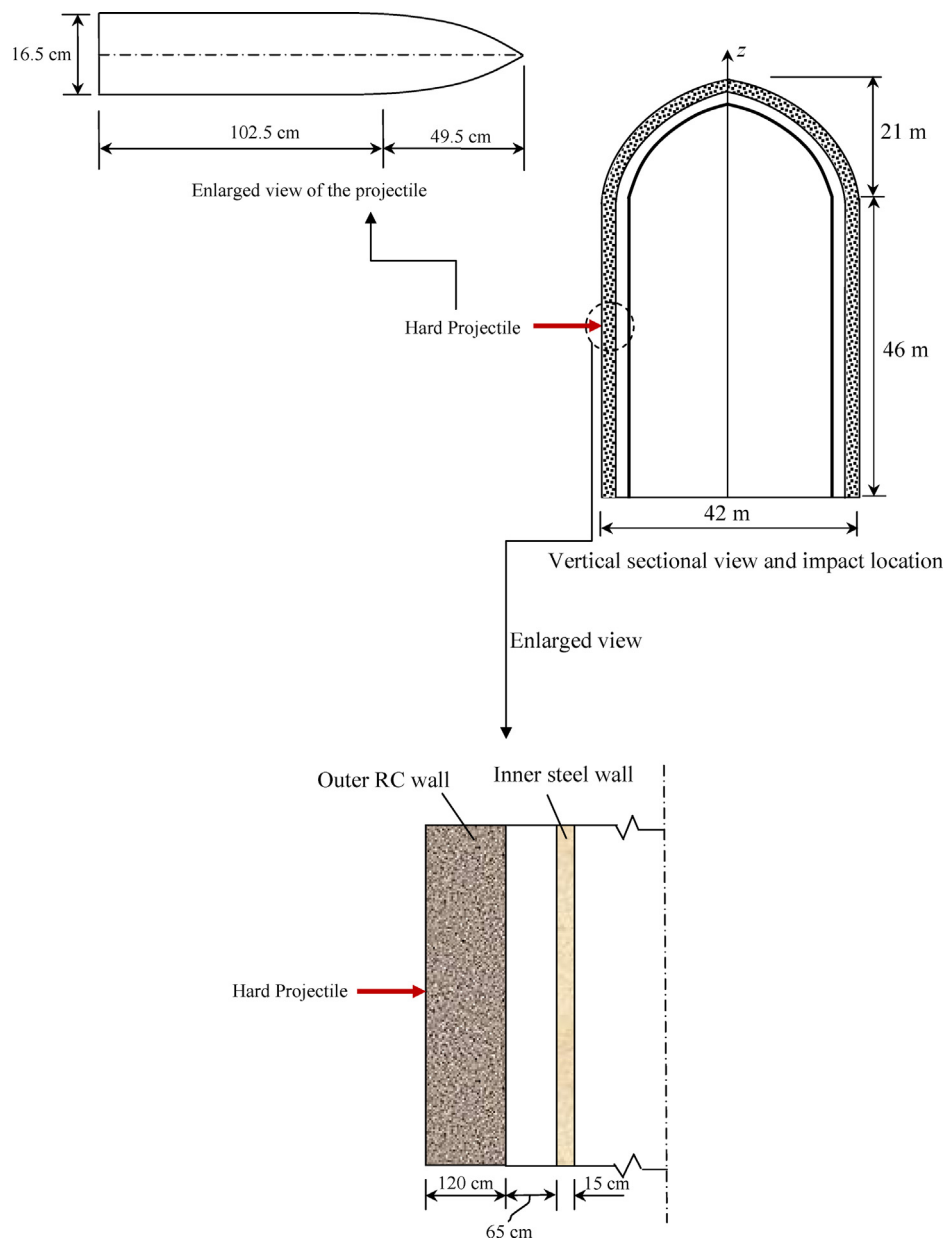


Fig. 1. Schematic details of outer concrete and inner steel walls of uniform thickness.

quantitative safety assessments were carried out against the impact of high velocity missiles, projectiles or even commercial planes. Such quantitative safety and reliability assessments are necessary in order to incorporate uncertainties involved in various design parameters related to the target (containment) and the impacting objects. Neglecting these uncertainties or considering them just in terms of qualitative safety factors may sometimes lead to a structure with substantially high probability of failure. As the consequences of containment structures failure are far severe than even the collapse of a 100 storey commercial building, it is very much necessary to propose a simple methodology for the reliability assessment of such containment structures against the impact of projectiles, missiles, etc. To propose a simple methodology for the reliability assessment of a double-wall containment structure against the impact of high velocity non-deforming projectiles is the subject of the current investigation. In the past, some limited works are reported on the reliability analysis of structures subjected to impact loads of projectiles, missiles and aircrafts on concrete

structures. Choudhury et al. (2002) presented a methodology for the reliability analysis of a buried concrete target against normal missile impact. The expressions for the depth of penetration in the buried target were derived. These equations were then employed for the reliability estimation. Design points, important for the probabilistic design, were located on the failure surface. Sensitivity analysis was carried out to study the influence of various random variables on target safety. Some parametric studies were included to obtain the results of field interest.

Siddiqui et al. (2003) presented a methodology for detailed reliability analysis of nuclear containment without metallic liners against aircraft crash. For this purpose, a nonlinear limit state function was derived using the violation of tolerable crack width as a failure criterion. The derived limit state requires the response of containment that was obtained from a detailed dynamic analysis of nuclear containment under the impact of a large size Boeing jet aircraft. Using this response in conjunction with limit state function, the reliabilities and probabilities of failures were obtained at

a number of vulnerable locations employing first-order reliability method (FORM). These values of reliability and probability of failure at various vulnerable locations were then used for the estimation of conditional and annual reliabilities of nuclear containment as a function of its location from the airport. To study the influence of the various random variables on containment reliability, the sensitivity analysis was performed.

Bhattacharya et al. (2013) described the methodology for developing a set of optimal reliability-based partial safety factors (PSFs) for the design of prestressed concrete inner containment shells in Indian NPPs under Main Steam Line Break (MSLB)/Loss of Coolant Accident (LOCA) conditions at two performance levels in flexure: cracking and collapse. A detailed numerical example on a typical 220 MWe Indian PHWR was provided to demonstrate the methodology.

Kim et al. (2013) carried out reliability analysis of nuclear containments considering long-term degradation in their tendon force. The gradual loss of tendon force was estimated from in-service inspection (ISI) data on nuclear containments in Korea. Using this data a degradation model for prestressing system was developed. Employing this model, reliability analyses were carried out for estimating the annual reliability of a typical nuclear containment against a limit state of through-wall cracking. The results of the analysis showed that the reliability is the lowest at 3/4 height of the studied containment wall.

Lo Frano and Forasassi (2011) numerically studied the effects and consequences of the energy transmitted to the outer containment walls due to a military or civil aircraft impact into a nuclear plant. They considered this event as a 'beyond design basis' event. A sensitivity analysis was also carried out considering the effects of different containment wall thickness and reinforced/prestressed concrete features. The obtained results were analyzed to check the NPP containment strength margins.

Pandey (1997) presented a quantitative reliability-based approach to evaluate the containment integrity in terms of the condition of bonded prestressing systems. The proposed approach utilized the results of lift-off, destructive, and flexural tests to update the probability distribution of prestressing force, and to revise the calculated reliability against through-wall cracking of containment elements. An acceptable criterion for the results of beam tests was established on the basis of maintaining adequate reliability throughout the service life of the containment.

Han and Ang (1998) suggested serviceability design load factors, and carried out the reliability assessment of RC containment structures. They developed probabilistic load factor for the limit state design of RC structures, and demonstrated how stochastic and advanced structural reliability methods can be systematically applied for the estimation of limit state probabilities of containment structures under stochastic dynamic loads such as accidental pressure and earthquakes loads. The serviceability limit states of crack failure that can cause emission of radioactive materials were suggested as critical failure criterion for RC containment structures. For this serviceability limit state, load factors were determined and target limit state probability was specified.

Penmetsa (2005) presented a methodology for the system reliability analysis that can determine the probability of the destruction of buried concrete targets using deep penetration weapons. Analytical equations for the depth of penetration and buckling strength of the missile were used to demonstrate an efficient system reliability analysis methodology. These equations were also used to determine the sensitivity of various parameters.

Siddiqui et al. (2009) carried out the reliability assessment of concrete targets subjected to impact forces due to striking projectiles. They identified various design parameters which can be judiciously chosen to achieve the desired reliability for concrete targets subjected to impact forces.

A detailed review of the literature on the reliability analysis of containment structures against possible impact of projectiles, missiles, etc. show that very limited studies are available on the reliability assessment of double-wall containment structures against the impact of projectiles. The aim of the present study is to present a simple methodology for the reliability assessment of double-wall containment structures against the impact of external non-deformable/hard projectiles.

## 2. Problem formulation

In the present study, a simple methodology for the reliability analysis of a double-wall containment structure against the impact of external hard projectiles is presented. The reliability analysis requires a limit state function which is a mathematical representation of a particular mode of failure. This function assumes a negative or zero value at the failure and a positive value when the structure is safe against that possible mode of failure. The probability of structural failure can then be defined as

$$P_f = P[g(\underline{x}) \leq 0] \quad (1)$$

where  $P_f$  represents probability of failure,  $g(\underline{x})$  is the limit state function and  $\underline{x}$  is the vector of basic random variables.

In order to derive the limit state function for the reliability analysis of a double-wall containment, a hard (non-deformable) sharp nose projectile is assumed to normally impact the outer RC wall with a velocity that it perforates the concrete wall and then hits the inner steel wall with its residual impact energy (or residual velocity). The double-wall containment is assumed to fail when the inner steel wall also gets perforated.

The failure of the steel wall/plate is assumed to occur when the projectile impact energy exceeds perforation energy of the steel wall. Keeping above points in view, if the perforation energy of the steel wall is  $E_{perf}$  and the projectile kinetic energy  $E_{proj}$  then the limit state function can be expressed as

$$g(\underline{x}) = E_{perf} - E_{proj} = E_{perf} - \frac{1}{2}MV_*^2 \quad (2)$$

where  $E_{proj}$  is the projectile energy;  $M$  the mass of the projectile;  $V_*$  the residual velocity of the projectile with which it hits the steel wall. The residual velocity is the velocity of impacting projectile after the perforation of the outer concrete wall.

From the above equation, it is obvious that the failure of the steel wall will occur if perforation energy ( $E_{perf}$ ) of the wall is equal to or less than the residual kinetic energy ( $= (1/2)MV_*^2$ ) of the impacting projectile.

In order to calculate the projectile impact energy with which it hits the inner steel wall, we need to estimate the residual velocity of the projectile. Thus, the formulation of limit state function involves the formulation of projectile energy at the exit of the outer RC wall and the perforation energy of the steel wall. The expressions for these two energies have been derived under the following assumptions and idealizations.

- The projectile is rigid, i.e. the deformation of projectile is negligible and only the concrete and steel wall deformations have been considered.
- Impact of projectile is normal to the concrete and the steel walls.
- The projectile does not carry any warhead and so no explosion has been considered.
- The loss of energy in the form of heat and sound has been neglected.

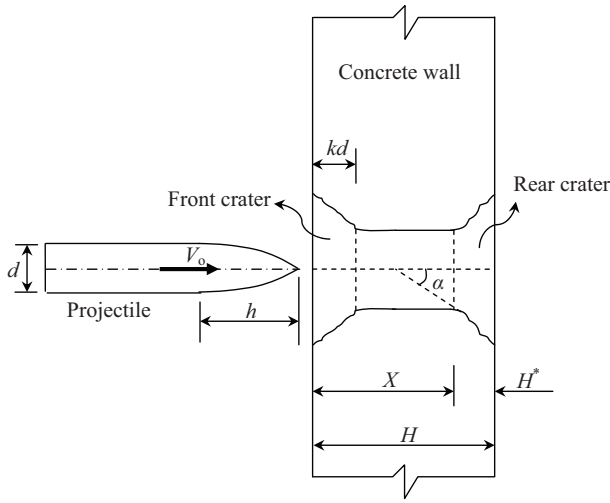


Fig. 2. Normal penetration of projectile into outer RC wall.

### 2.1. Perforation energy of the steel wall

The energy required to perforate a steel wall was obtained by the empirical equation originally proposed by Thomson (Thomson, 1954; Corbett et al., 1996) for steel plates. This equation can be expressed as

$$E_{perf} = \pi d^2 t \left( 0.125Y + 0.0625 \rho_s C_E \left( \frac{V_* d}{h} \right)^2 \right) \quad (3)$$

where  $E_{perf}$  is the perforation energy;  $d$  the diameter of the aft body of projectile;  $t$  the thickness of steel wall;  $Y$  the yield stress of the steel wall;  $\rho_s$  the density of the steel wall;  $h$  the nose length of the projectile;  $C_E$  the constant = 1 for a conically tipped projectile and  $C_E = 1.86$  for an ogival-headed projectile. Sodha and Jain (1958) subsequently corrected the analysis for the ogival-headed projectile, giving a new value of  $C_E = 0.62$  (Corbett et al., 1996).

### 2.2. Residual velocity of the projectile

The process of penetration of projectile into concrete wall involves the initial cratering, tunneling and rear cratering (Fig. 2). When a rigid projectile of mass  $M$ , diameter  $d$ , and a general convex nose shape impacts a RC target of thickness  $H$  with initial velocity  $V_0$ , penetrates through the concrete target, the formation of initial crater can be assumed as a frustum of cone with depth  $kd$  (Fig. 2). Here  $k$  is a dimensionless parameter which can be estimated using the expression proposed by Li and Chen (2003):

$$k = 0.707 + \frac{h}{d} \quad (4)$$

where  $h$  is the nose length of the projectile, and  $d$  the diameter of the projectile.

During the penetration, the concrete offers axial resisting force on the projectile nose. This resisting force  $F$  can be estimated using the empirical equations proposed by Forrestal et al. (1994):

When projectile is in the front crater region, i.e. when  $X/d \leq k$ :

$$F = c \frac{X}{d} \quad (5)$$

However, when the projectile has crossed the front crater, i.e. when  $X/d > k$ :

$$F = \frac{\pi d^2}{4} (Sf'_c + N^* \rho_c V^2) \quad (6)$$

where  $X$  is the instantaneous penetration depth;  $c$  an empirical constant;  $V$  the instantaneous projectile velocity;  $\rho_c$  the density of the

concrete target;  $S$  the dimensionless parameter which is a function of unconfined compressive strength of concrete =  $82.6f'_c{}^{-0.544}$  (Frew et al., 1998), where  $f'_c$  the unconfined compressive strength of concrete in MPa;  $N^*$  the projectile nose shape factor.

The factor  $c$  used in Eq. (5) can be estimated using the formulation proposed by Chen et al. (2004) as given below:

$$c = \frac{(\pi d^2 Sf'_c / 4k)(1 + (I/N))}{1 + (k\pi / 4N)} \quad (7)$$

Here  $I$  and  $N$  are impact and geometry functions, respectively. These equations can be estimated using (Chen et al., 2008)

$$I = \frac{MV_0^2}{d^3 Sf'_c} \quad (8)$$

$$N = \frac{M}{\rho_c d^3 N^*} \quad (9)$$

where  $M$  is the mass of the projectile. For ogive and conical nose projectiles,  $N^*$  can be estimated using (Chen and Li, 2002):

$$N^* = \frac{1}{3\psi} - \frac{1}{24\psi^2} \quad \text{for ogive nose and} \quad (10)$$

$$N^* = \frac{1}{1 + 4(h/d)^2} \quad \text{for conical nose} \quad (11)$$

Here  $\psi$  is the caliber-radius-head (CRH) of ogive nose (Appendix).

Integration of Eqs. (5) and (6) after substituting  $c$  from Eq. (7) yields a dimensionless maximum depth of penetration, which can be simplified for sharp and slender projectiles into the following equations (Chen et al., 2008):

$$\frac{X}{d} = \sqrt{\frac{4k}{\pi}} I \quad \text{for } \frac{X}{d} \leq k \quad \text{or} \quad I \leq \frac{k\pi}{4} \quad (12a)$$

$$\frac{X}{d} = \frac{2I}{\pi} + \frac{k}{2} \quad \text{for } \frac{X}{d} > k \quad \text{or} \quad I > \frac{k\pi}{4} \quad (12b)$$

If the concrete wall is perforated, the velocity after perforation is called the residual velocity ( $V_*$ ), which can be computed by (Chen et al., 2004, 2008)

$$V_* = V_0 - V_{BL} \quad \text{for } \chi \leq \chi_c \quad (13a)$$

$$V_* = \sqrt{V_0^2 - V_{BL}^2} \quad \text{for } \chi > \chi_c \quad (13b)$$

where  $V_{BL}$  is the ballistic limit velocity, which is defined as the minimum velocity for the perforation of RC wall. The ballistic limit equations were formulated by Chen et al. (2008) for an RC target as

$$V_{BL} = \sqrt{\frac{\pi d^3 Sf'_c}{4kM} \left( \chi - \frac{H_{BL}^*}{d} \right)} \quad \text{for } \chi \leq \chi_c \quad (14a)$$

$$V_{BL} = \sqrt{\frac{\pi d^3 Sf'_c}{2M} \sqrt{\left( \chi - \chi_c + \frac{k}{d} \right)}} \quad \text{for } \chi > \chi_c \quad (14b)$$

where  $\chi$  is the dimensionless thickness of concrete such that  $\chi = H/d$  and  $H_{BL}^*$  the thickness of conical plug at the ballistic limit;  $\chi_c$  the dimensionless critical thickness of concrete target which can be estimated using (Chen et al., 2008):

$$\chi_c = \frac{\sqrt{(1 + \Theta \tan \alpha)^2 + (\sqrt{3S} - 4\Theta) \tan \alpha} - (1 + \Theta \tan \alpha)}{2 \tan \alpha} \quad (15)$$

where  $\Theta$  is the dimensionless number which is defined as

$$\Theta = \sqrt{3} \chi p_s \frac{f_s}{f'_c} \sin \alpha \quad (16)$$

where  $p_s$  is the reinforcement ratio;  $f_s$  the uniaxial tensile strength of reinforcing bars;  $\alpha$  the cone slope angle of rear crater of concrete target (Fig. 2).

$H_{BL}^*$  can be estimated using

$$\frac{H_{BL}^*}{d} = \frac{\sqrt{(1 + \Theta \tan \alpha)^2 + (\sqrt{3S} - 4\Theta) \tan \alpha - (1 + \Theta \tan \alpha)}}{2 \tan \alpha} \quad \text{for } \frac{X}{d} > k \quad (17a)$$

$$\frac{H_{BL}^*}{d} = \frac{\sqrt{(1 + \Theta \tan \alpha)^2 + [\sqrt{3S}(X/kd) - 4\Theta] \tan \alpha - (1 + \Theta \tan \alpha)}}{2 \tan \alpha} \quad \text{for } \frac{X}{d} \leq k \quad (17b)$$

Substituting the expressions of perforation energy and the residual velocity of the projectile from Eqs. (3) and (13a, 13b) into Eq. (2), we have

$$g(\underline{x}) = \pi d^2 t \left[ 0.125Y + 0.0625\rho_s C_E \left( \frac{(V_0 - V_{BL})d}{h} \right)^2 \right] - \frac{1}{2}M(V_0 - V_{BL})^2 \quad \text{for } \chi \leq \chi_c \quad (18a)$$

$$g(\underline{x}) = \pi d^2 t \left[ 0.125Y + 0.0625\rho_s C_E \left( \frac{(V_0 - V_{BL})d}{h} \right)^2 \right] - \frac{1}{2}M(V_0^2 - V_{BL}^2) \quad \text{for } \chi > \chi_c \quad (18b)$$

As in the above limit state equations, the variables  $V_0, d, f'_c, \rho_c, f_y, h, M, \rho_s, Y, H, t, p_s$  have significant inherent uncertainties; they will be considered as random variables in the subsequent reliability analysis. Arranging these variables in vector form leads to

$$\underline{x} = (V_0, d, f'_c, \rho_c, f_y, h, M, \rho_s, Y, H, t, p_s) \quad (19)$$

Here  $\underline{x}$  is the vector of basic random variables.

It is worth mentioning that, in the above formulation, the effect of friction on the length of the cylindrical portion is neglected. This is due to the fact that after the full penetration of the projectile nose through outer concrete wall, the created cavity will expand a little. Due to this little expansion of the concrete, the cylindrical portion of the projectile will no more be in contact with the concrete wall and the friction on the length of the cylindrical portion can be neglected (Chen et al., 2004, 2008; Choudhury et al., 2002; Forrestal et al., 1994).

The matrix of the dimensions for the random variables involved in the above equation is

$$\Omega = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & -3 & -1 & 1 & 0 & -3 & -1 & 1 & 1 & 0 \\ -1 & 0 & -2 & 0 & -2 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \end{bmatrix}$$

The columns of the above matrix correspond to the variables in the order in which they appear in Eq. (19) and the rows of the matrix correspond to the three fundamental dimensions viz.  $M$  (mass),  $L$  (Length) and  $T$  (Time). The number of fundamental dimensions involved in the model is three and the rank of the above dimensional matrix is also 3, thus according to the Buckingham-PI theorem, the number of dimensionless parameters required for modeling would be 9 ( $12 - 3 = 9$ ). As the total number of dimensional variables is 11, there is no much advantage of converting the variables into dimensionless form. It is due to this reason that in the present study, the raw variables are preferred over dimensionless variables.

Having derived the limit state functions, the next step is the assessment of probability of failure (also called risk) and reliability (measured in terms of reliability index  $\beta$ ) of the double-wall containment structure against the normal impact of the projectile. For this purpose, Monte Carlo Simulation technique (Nowak and Collins, 2000) has been employed. A brief description of this method is given in the following section.

### 2.3. Monte Carlo simulation

Monte Carlo simulation consists of drawing samples of the basic random variables according to their probabilistic characteristics and then feeding them into the limit state function. It is known that the failure occurs when  $g(\underline{x}) < 0$ ; therefore an estimate of the probability of failure  $P_f$  can be found by

$$P_f = \frac{n_f}{n} \quad (20)$$

where  $n_f$  is the number of simulation cycles in which,  $g(\underline{x}) < 0$ , and  $n$  is the total number of simulation cycles. As  $n$  approaches infinity, the  $P_f$  approaches to the true probability of failure.

Having known the probability of failure, reliability index  $\beta$  can be determined using  $\beta = -\Phi^{-1}(P_f)$ , where  $P_f$  is the probability of failure and  $\Phi^{-1}(\cdot)$  is the inverse of standard normal distribution function.

The accuracy of Eq. (20) can be evaluated in terms of its variance. For a small number of simulation cycles, the variance of  $P_f$  can be quite large. Consequently, it may require a large number of simulation cycles to achieve a specified accuracy. The variance of the estimated probability of failure can be computed by assuming each simulation cycle to constitute a Bernoulli trial. Therefore, the number of failures in  $n$  trials can be considered to follow a binomial distribution. Then the variance of the estimated probability of failure can be computed approximately as

$$\text{Var}(P_f) = \frac{(1 - P_f)P_f}{n} \quad (21)$$

It is recommended to measure the statistical accuracy of the estimated probability of failure by computing its coefficient of variation (COV) as

$$\text{COV}(P_f) \cong \frac{\sqrt{((1 - P_f)P_f)/n}}{P_f} \quad (22)$$

The smaller the COV, the better is the accuracy of the estimated probability of failure. It is evident from Eqs. (21) and (22) that as  $n$  approaches infinity,  $\text{Var}(P_f)$  and  $\text{COV}(P_f)$  approaches zero. However, for all practical purposes, the number of simulation cycles for which  $\text{COV}(P_f)$  approaches less than 5% may be considered as an appropriate number of simulation cycles (Nowak and Collins, 2000). In the present study, 1 million simulations were used to carry out the Monte Carlo simulation for estimating the probabilities of failures.

### 3. Numerical study

In order to illustrate the above procedure for carrying out the reliability analysis of double-wall containment structures against the normal impact of hard projectiles, an idealized double-wall containment was selected as shown in Fig. 1. A hard (non-deformable) ogival nose shape, 1.52 m long projectile of nominal mass of 182 kg, nose length of 0.495 m and CRH of 9.25 was employed to hit the outer concrete wall of the containment (Fig. 1). A gap of 65 cm exists between the outer and inner walls as shown in Fig. 1. This gap is selected as 65 cm to keep it more than the nose length of the projectile. The impact velocity was taken as a percentage of the nominal ballistic limit  $V_{BL}$  of outer 1.2 m thick RC wall. The nominal ballistic limit of this wall was obtained from the deterministic analysis for different steel ratios as shown in Fig. 3. For the present containment, having steel ratio of 1.1%, ballistic limit of the outer concrete wall was obtained as 290 m/s. The statistical characteristics and probability distributions of the random variables (Eq. (19)), required for the reliability analysis, were judiciously selected as shown in Table 1. The references used for the selection of the coefficient of variation (COV) and the probability distributions of various random variables are given in the last column of this table. For some of the



**Table 1**  
Random variables considered for the reliability analysis of the double-wall containment structure.

Random variable	Nominal	Bias factor	COV	Distribution	Reference
<i>Concrete wall</i>					
Concrete strength, $f'_c$ (MPa)	40	0.9	0.10	Lognormal	Penmetsa (2005)
Reinforcement ratio, $p_s$ (%)	1.1	0.9	0.10	Normal	Assumed
Uni-axial tensile strength of reinforcing bars, $f_y$ (MPa)	420	0.9	0.10	Lognormal	Assumed
Thickness of concrete target, $H$ (m)	1.2	1.0	0.05	Normal	Siddiqui et al. (2002)
Concrete density, $\rho_c$ (kg/m <sup>3</sup> )	2440	0.95	0.10	Lognormal	Choudhury et al. (2002)
<i>Steel wall</i>					
Yield strength of the steel wall, $Y$ (MPa)	420	0.95	0.05	Normal	Assumed
Steel density, $\rho_s$ (kg/m <sup>3</sup> )	7850	0.95	0.10	Lognormal	Assumed
Thickness of steel wall, $t$ (mm)	150	1.00	0.03	Normal	Assumed
<i>Projectile</i>					
Nose length of the projectile, $h$ (mm)	495	1.00	0.025	Normal	Penmetsa (2005)
Diameter of the projectile, $d$ (mm)	165	1.05	0.05	Normal	Penmetsa (2005)
Mass of the projectile, $M$ (kg)	182	1.10	0.05	Lognormal	Penmetsa (2005)
Impact velocity, $V_0$ (m/s)	Variable	1.00	0.10	Extreme type-1	Choudhury et al. (2002)

COV: coefficient of variation.

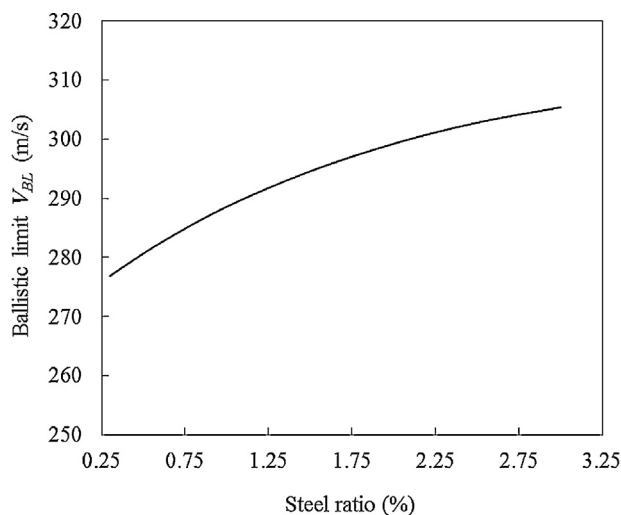
parameters, the desired statistical data were not directly available in the literature. They were thus assumed, keeping in mind their practical range and widely used probability distributions. In this table, the bias factor represents the ratio of mean to the nominal value. When bias factor is one, it indicates that the nominal value is the same as the mean value. In general, for resistance related variables, bias factor is considered greater than one. Bias factor of less than 1.0 is generally assumed for the load related variables. The value of COV is a measure of degree of uncertainty.

#### 4. Discussion of results

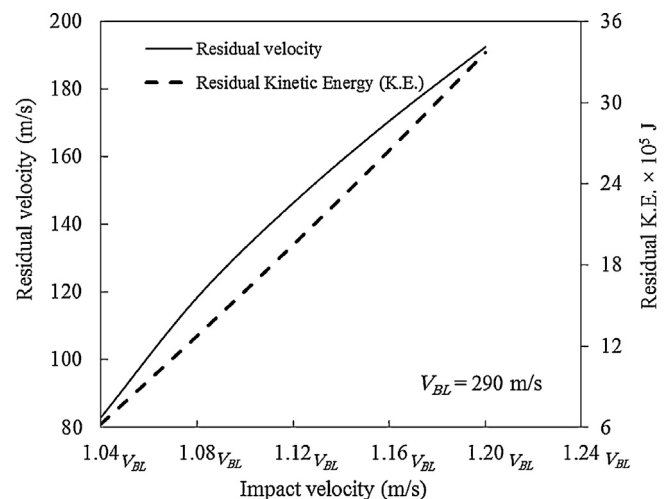
Employing the data presented in Table 1 and using the Monte Carlo simulation technique, the reliability analysis of the containment was carried out for different impact velocities taken as a percentage of the nominal ballistic limit of the outer RC wall (=290 m/s). The results are shown in Table 2, which illustrates that with a little increase in the impact velocity, there is a substantial increase in the failure probability. This is an expected trend as high impact velocity will impart high kinetic energy to the projectile which in turn will impact the inner steel wall with a higher residual velocity (or residual kinetic energy). This table clearly illustrates that an increase of only 10 m/s (~4%) in the residual velocity causes about 30–40% increase in the residual kinetic energy of the projectile. Fig. 4 shows the variation

of nominal residual velocity and residual kinetic energy of the projectile with the striking velocity.

Fig. 5 shows that under given uncertainties, if the impact velocity is less than 65% of the nominal ballistic limit (i.e.  $0.65V_{BL}$ ), the containment is sufficiently reliable as for this impact velocity the reliability index is above 3. Any important structure with reliability index above 3.0 is generally considered sufficiently reliable. In the present study, the desired reliability index of 3.0 and 3.5 are considered as these are the two typical desired values for the structures of importance (Siddiqui et al., 2002, 2003; Choudhury et al., 2002). When impact velocity is more than 90% of the ballistic limit (i.e.  $0.90V_{BL}$ ), failure probability is quite high as the reliability index is less than 1.0 which is an indication that the containment is not safe as desired. In other words, the present double-wall containment is safe enough against the impact of the projectile if the projectile nominal impact velocity is less than 0.65 times the nominal ballistic limit ( $V_{BL}$ ) of containment outer concrete wall. It is worth mentioning, although the nominal impact velocity is less than the nominal ballistic limit of outer wall, there is a finite probability of steel wall penetration. This is due to the fact that when one million impact velocities were simulated, due to the uncertainties involved, a good number of impact velocities had values more than the nominal ballistic limit. Owing to this reason, the steel wall has finite probability of failure even in those cases when the nominal impact velocity is less than the nominal ballistic limit ( $V_{BL}$ ) of the outer RC wall.



**Fig. 3.** Variation of nominal ballistic limit of the outer RC wall with the steel ratio.



**Fig. 4.** Variation of nominal residual velocity and residual kinetic energy of the projectile with impact velocity.

**Table 2**  
Striking velocity and change in the residual kinetic energy of the projectile.

Striking velocity, $V_0$ of the projectile	Residual velocity of the projectile (m/s)	Residual kinetic energy of the projectile (kJ)	Change in the residual kinetic energy of the projectile (%)
$1.04V_{BL}$	83	626.6	–
$1.08V_{BL}$	118	1275.7	103.6
$1.12V_{BL}$	146	1949.1	211.1
$1.16V_{BL}$	171	2646.9	322.4
$1.20V_{BL}$	192	3369.7	437.8

$V_{BL}$ : nominal ballistic limit of the outer RC wall.

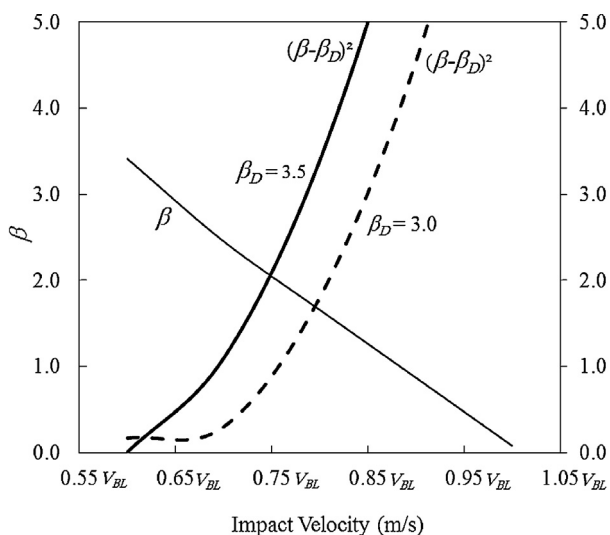
Fig. 5 illustrates the variation of reliability index with the impact velocity expressed in terms of the percentage of outer concrete wall ballistic limit. This figure shows that when the impact velocity is close to  $0.65V_{BL}$ ,  $(\beta - \beta_D)^2$  is close to zero for the desired reliability index of 3.0–3.5. This is an indication that the containment is “safe enough” if the striking velocity of the impacting projectile is less than  $0.65V_{BL}$ .

#### 4.1. Sensitivity study

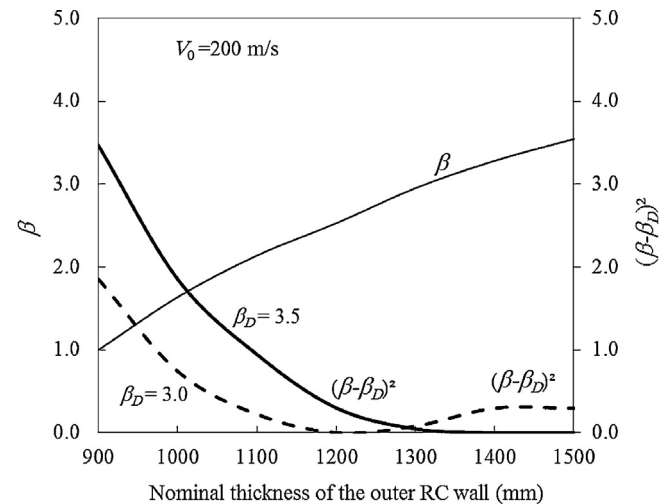
In the present section, a few sensitivity analyses were carried out to obtain the results of practical interest. For the sensitivity study of a variable, its nominal value was varied to study its effect on failure probability and reliability index of the double-wall containment. The impact velocity of the projectile was taken as 200 m/s and all other variables were taken same as shown in Table 1. The sensitivity study was carried out to study the influence of the variables on the containment reliability.

##### 4.1.1. Effect of the concrete wall thickness

Fig. 6 shows that as the thickness of the outer concrete wall increases, reliability of the containment also increases. This is an expected trend as the concrete thickness increases the residual velocity of the projectile decreases, which reduces the failure probability of the inner steel wall. Fig. 6 also shows that a little change in the thickness of the concrete wall can alter the reliability substantially. This figure clearly illustrates that an increase of 200 mm in the concrete thickness can increase the containment reliability index approximately by 1. This change in the reliability of the containment is due to the change in the residual kinetic energy of the projectile.



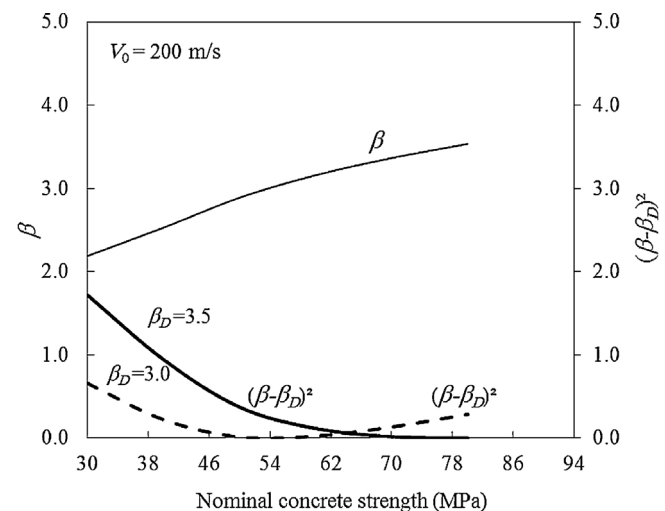
**Fig. 5.** Variation of containment reliability with impact velocity.



**Fig. 6.** Variation of the containment reliability with the thickness of the outer RC wall.

##### 4.1.2. Effect of the concrete strength

Fig. 7 shows that as the concrete strength increases, reliability of the containment increases. This is due to the fact that an increase in the concrete strength will decrease the residual velocity of the projectile which will increase the overall reliability of the containment. Fig. 7 also shows that for the desired reliability index of 3.0, the strength of the concrete should not be less than 54 MPa and for achieving the desired reliability of 3.5, the strength of concrete should be about 70 MPa. In other words, a minimum concrete strength of 54 MPa is desirable for achieving the desired safety of the containment against the impact of the present projectile.



**Fig. 7.** Variation of the containment reliability with the concrete strength.

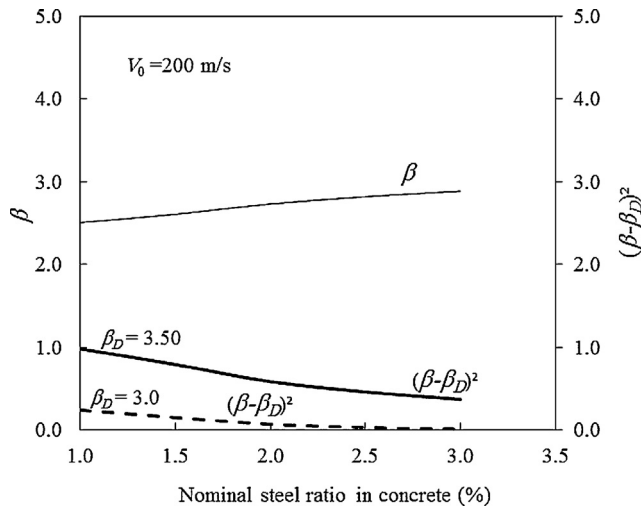


Fig. 8. Variation of the containment reliability with the nominal steel ratio in concrete.

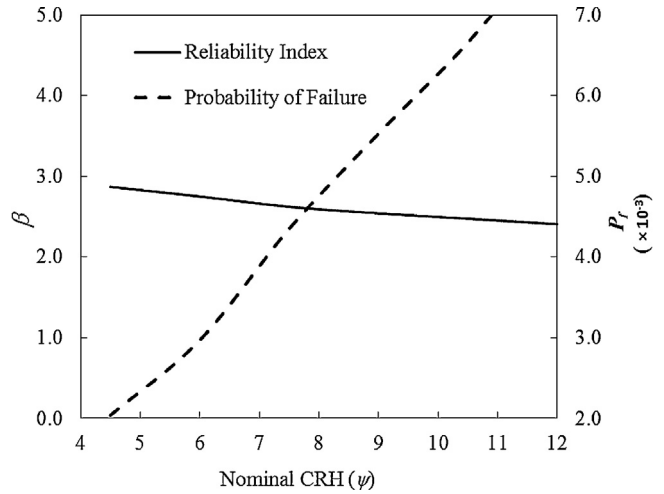


Fig. 10. Variation of the containment reliability with the CRH of the projectile.

dependence of wall perforation energy on the yield strength ( $Y$ ) of the steel wall (Eq. (3)).

#### 4.1.5. Effect of CRH of the projectile

Fig. 10 shows the variation of the reliability index with change in the caliber radius head (CRH) of ogival nose-shaped projectiles. This figure shows that as the CRH of the projectile nose increases, the failure probability increases. It is due to the fact that as CRH increases, the shape of the nose becomes more pointed which makes the penetration of projectile easier. Thus it can be concluded that a projectile with higher CRH is more dangerous to the containment than a projectile with other parameters same, but lesser CRH. When a projectile of higher CRH impacts the containment, it increases the failure probability of the containment sharply. It is worth mentioning that although in the formulation, the nose length ( $h$ ) was considered as random variable (Eq. (19)), but the sensitivity study is presented for CRH as it is more appropriate to express nose shape of ogival projectile by CRH. Nose length ( $h$ ) and CRH ( $\psi$ ) are related to each other by a simple equation,  $h = \sqrt{\psi^2 d^2 - (\psi d - 0.5d)^2}$ , where  $d$  is the projectile diameter. The derivation of this equation is shown in Appendix.

## 5. Conclusions

In the present study, a simple probabilistic procedure based on Monte Carlo simulation technique was presented to study the reliability of double-wall containment against impact of ogive nose shape projectiles. The safety of the containment was correlated with the ballistic limit of the outer RC wall. Following are the conclusions which can be drawn from the present numerical study:

- The present double-wall containment is safe enough against the impact of the projectile if the projectile nominal impact velocity is less than 65% of the nominal ballistic limit  $V_{BL}$  of the containment outer-wall.
- An increase of only 10 m/s ( $\sim 4\%$ ) in the residual velocity causes about 30–40% increase in the residual kinetic energy of the projectile. Owing to this reason, a little change in the impact velocity may cause a phenomenal change in the containment reliability.
- As expected, with an increase in the impact velocity of the projectile there is a sharp decrease in the reliability of the double-wall containment.
- Sensitivity analysis of important variables was carried out by varying their nominal values and keeping the statistical values of all other variables constant. The outer wall of the

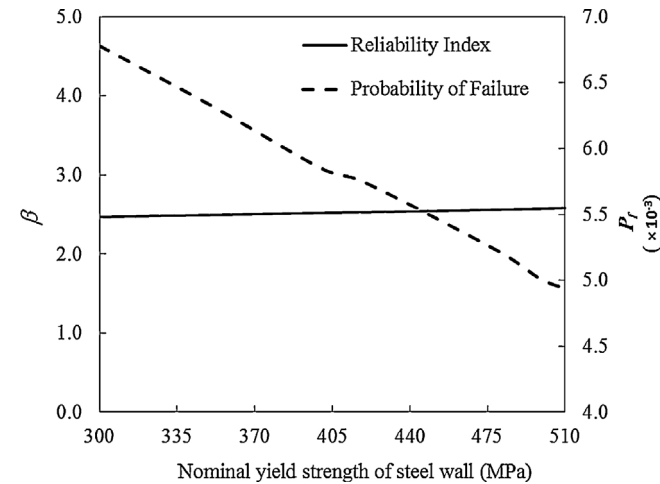


Fig. 9. Variation of the containment reliability with the strength of the steel wall.

#### 4.1.3. Effect of reinforcement ratio

Fig. 8 shows that as the steel ratio in outer concrete wall increases, reliability of the containment also increases. This can be attributed to the increased strength of the outer concrete wall due to increase in the steel ratio. An increase in the outer wall strength decreases the residual velocity of the projectile. A reduction in the residual velocity decreases the probability of inner steel wall perforation which consequently increases the reliability of the containment. Fig. 8 clearly shows that with the steel ratio variation, desired reliability index of 3.0 can be achieved by keeping the steel ratio about 2.5%. However, only by changing the steel ratio in a practical range, a reliability index of 3.5 is not achievable. In such a situation some other variables have to be also varied to achieve desired reliability of 3.5.

#### 4.1.4. Effect of yield strength of the steel wall

Fig. 9 shows that as the yield strength of the steel wall increases, probability of containment failure decreases almost linearly. The decrease in the containment failure probability, due to increase in the yield strength of the steel wall, can be attributed to a linear



containment was assumed to be impacted by the projectile with 200 m/s velocity. The results of this sensitivity study are:

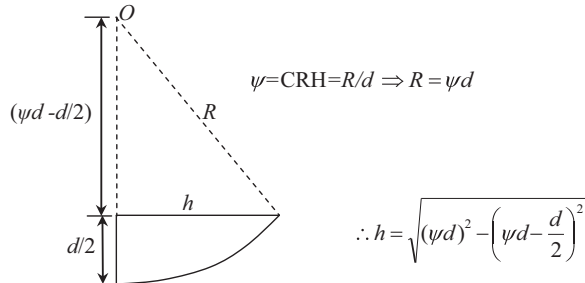
- A change in the concrete wall thickness can alter the reliability substantially. An increase of 200 mm in the concrete thickness can increase the containment reliability index approximately by 1.
- As the concrete strength increases, reliability of the containment increases. In order to achieve the desired reliability index of 3.0, the strength of the concrete should not be less than 54 MPa, and for achieving the desired reliability of 3.5, the strength of the concrete should be about 70 MPa.
- As the steel ratio in the outer concrete wall increases, probability of containment failure decreases. By varying the steel ratio alone, desired reliability index of 3.0 can be achieved if the steel ratio is around 2.5%. However, only by changing steel ratio in a practical range, a reliability index of 3.5 is not achievable. To achieve this reliability some other parameters have to be also varied along with steel ratio.
- As the yield strength of the steel wall increases, probability of containment failure decreases almost linearly.
- As the CRH of the projectile nose increases, the failure probability of the containment increases.

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### Appendix.

Relation between CRH ( $\psi$ ) and nose length ( $h$ ) of the projectile:



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